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Massive supermultiplets with spin 3/2

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Abstract: In this paper we construct massive supermultiplets out of appropriate set of massless ones in the same way as massive spin s particle could be constructed out of massless spin $s, s-1, \ldots$ ones leading to gauge invariant description of massive particle. Mainly we consider massive spin 3/2 supermultiplets in a flat $d=4$ Minkowski space both without central charge for $N=1,2,3$ as well as with central charge for $N=2,4$. Besides, we give two examples of massive $N=1$ supermultiplets with spin 3/2 and 2 in AdS_4 space.

Keywords: Extended Supersymmetry, Supersymmetry Breaking.

Contents

1. Introduction

In a flat space-time massive spin s particles in a massless limit decompose into massless spin s, $s-1$, ... ones. This, in particular, leads to the possibility of gauge invariant description of massive spin s particles, where massless spin s field plays the role of "main" gauge field, while the lower spin fields play the roles of Goldstone fields that have to be "eaten" in the process of spontaneous symmetry breaking to make main field massive. Such approach to description of massive particles became rather popular last times, e.g. [1]–[13].

In the supersymmetric theories all particles must belong to some supermultiplet, massive or massless. The same reasoning on the massless limit means that massive supermultiplets could (should) be constructed out of the massless ones in the same way as massive particles out of the massless ones. Because supersymmetry is a very restrictive symmetry even construction of free massive supermultiplets could give very usefull and important information on the structure of full interacting theories, where spontaneous (super)symmetry breaking leading to the appearance of such massive supermultiplets could occur.

In supergravities partial super-Higgs effect $N \to N-k$, when part of the supersymmetries remains unbroken, must unavoidably leads to the appearance of k massive spin 3/2 supermultiplets, corresponding to the unbroken $N-k$ supersymmetries. The main subject of our paper is the construction of massive spin 3/2 supermultiplets out of the massless ones. Namely, we consider $N = 1, 2, 3$ supermultiplets without central charge, as well as $N = 2, 4$ supermultiplets with central charge (for classification of massless and massive supermultiplets see e.g. [14]). We will not consider one more possible case — $N = 6$ supermultiplet with central charge, because it could hardly have phenomenological interest, though it could be constructed in the same way as well. Besides, we consider two examples of massive supermultiplets in (A)dS space, namely $N = 1$ spin 3/2 and spin 2 ones.

The paper is organized as follows. In the next section we start with the simplest case — massive $N = 1$ spin 3/2 supermultiplet [15–18]. This supermultiplets is known for a long time, but it is very usefull to display the general technics for construction of massive supermultiplets we will heavily use in what follows. There is no strict definition of what is mass in (Anti) de Sitter space and indeed rather different definitions there exist in the literature. So we add small section devoted to the discussion of this subject (and in particular the so called forbidden mass regions, see e.g. [19–21]) using massive spin 3/2 particle in AdS space as an example. Then, in the next two sections, we consider massive spin 3/2 [22] and massive spin 2 supermultiplets [18, 23–25] in AdS space. Our results here show rather interesting differences between supermultiplets in flat and AdS spaces, as well as between supermultiplets with integer and half-integer superspins. Also, massive spin 2 supermultiplet in AdS shows one more example of the flat space limit — massless limit ambiguity, which is well known for the massive spin 2 [26–28] and spin 3/2 [29–31] particles.

Then we return back to the flat Minkowski space and in the following four sections we systematically consider massive spin 3/2 supermultiplets with $N = 2$ and $N = 3$ supersymmetry without central charge as well as $N = 2$ and $N = 4$ supermultiplets with central charge. In all cases exactly as in $N = 1$ case it turns out crucial for the whole construction to make duality transformations mixing different supermultiplets containing vector fields.

2. $N = 1$ supermultiplet in flat space

Let us start with the simplest case — massive $N = 1$ supermultiplet [15–18] in flat space-time. Such multiplet contains massive particles with spins $(3/2, 1, 1', 1/2)$, all with equal masses. In the massless limit massive spin 3/2 particle decompose into massless spin 3/2 and 1/2 ones in the same way as massive spin 1 particle into massless spin1 and spin 0 ones. As a result in the massless limit our massive supermultiplet gives three massless supermultiplets:

$$\begin{pmatrix} & 3/2 & \\ 1 & & 1' \\ & 1/2 & \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 3/2 \\ 1 \end{pmatrix} \oplus \begin{pmatrix} 1' \\ 1/2 \end{pmatrix} \oplus \begin{pmatrix} 1/2 \\ 0, 0' \end{pmatrix}$$

We denote appropriate fields as (Ψ_μ, A_μ), (B_μ, ρ) and (χ, φ, π), correspondingly. We start with the massless Lagrangian being the sum of kinetic terms for all these fields:

$$\mathcal{L}_0 = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{\Psi}_\mu\gamma_5\gamma_\nu\partial_\alpha\Psi_\beta + \frac{i}{2}\bar{\rho}\hat{\partial}\rho + \frac{i}{2}\bar{\chi}\hat{\partial}\chi - \frac{1}{4}{A_{\mu\nu}}^2 - \frac{1}{4}{B_{\mu\nu}}^2 + \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}(\partial_\mu\pi)^2 \quad (2.1)$$

which is invariant under three local gauge transformations:

$$\delta\Psi_\mu = \partial_\mu \xi, \qquad \delta A_\mu = \partial_\mu \lambda, \qquad \delta B_\mu = \partial_\mu \tilde{\lambda}$$

It is very important that the massive supermultiplet must contains vector and axial-vector particles and not two vector or two axial-vector ones. This, in turn, opens the possibility to make dual transformation mixing two supermultiplet, namely (Ψ_μ, A_μ) and (B_μ, ρ) ones. Thus, the most general supertransformations leaving the massless Lagrangian invariant have the form:

$$\begin{aligned}
\delta\Psi_\mu &= -\frac{i}{2\sqrt{2}}\sigma^{\alpha\beta}[\cos(\theta)A_{\alpha\beta} - \sin(\theta)B_{\alpha\beta}\gamma_5]\gamma_\mu\eta \\
\delta A_\mu &= \sqrt{2}\cos(\theta)(\bar{\Psi}_\mu\eta) + i\sin(\theta)(\bar{\rho}\gamma_\mu\eta) \\
\delta B_\mu &= \sqrt{2}\sin(\theta)(\bar{\Psi}_\mu\gamma_5\eta) + i\cos(\theta)(\bar{\rho}\gamma_\mu\gamma_5\eta) \\
\delta\rho &= -\frac{1}{2}\sigma^{\alpha\beta}[\sin(\theta)A_{\alpha\beta} + \cos(\theta)B_{\alpha\beta}\gamma_5]\eta \\
\delta\chi &= -i\hat{\partial}(\varphi + \gamma_5\pi)\eta \quad \delta\varphi = (\bar{\chi}\eta) \quad \delta\pi = (\bar{\chi}\gamma_5\eta)
\end{aligned} \tag{2.2}$$

Now we have to add mass terms for all fields as well as appropriate corrections for the fermionic supertransformations. For this purpose we, first of all, must identify Goldstone fields which have to be eaten by gauge fields making them massive. For bosonic fields the choice is unambiguous — scalar field φ for vector A_μ and pseudo-scalar π for axial-vector B_μ. Thus, we add the following mass terms:

$$\mathcal{L}_m = -mA^\mu\partial_\mu\varphi - mB^\mu\partial_\mu\pi + \frac{m^2}{2}{A_\mu}^2 + \frac{m^2}{2}{B_\mu}^2 \tag{2.3}$$

As for the spin 3/2 particle Ψ_μ, we have two spinor fields ρ and χ which could serve as a Goldstone one, so we consider the most general possible mass terms:

$$\frac{1}{m}\mathcal{L}_m = \frac{1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + ia_1(\bar{\Psi}\gamma)\rho + ia_2(\bar{\Psi}\gamma)\chi + a_3\bar{\rho}\rho + a_4\bar{\rho}\chi + a_5\bar{\chi}\chi \tag{2.4}$$

Then the requirement that the total Lagrangian be invariant under (corrected) supertransformations fixes the mixing angle θ as well as all unknown coefficients:

$$\sin(\theta) = \cos(\theta) = \frac{1}{\sqrt{2}}, \quad a_1 = -\frac{1}{\sqrt{2}}, \quad a_2 = 1, \quad a_3 = 0, \quad a_4 = -\sqrt{2}, \quad a_5 = \frac{1}{2}$$

Moreover, this requirement unambiguously fixes the structure of appropriate corrections for fermionic supertransformations:

$$\begin{aligned}
\frac{1}{m}\delta\Psi_\mu &= \left[A_\mu + B_\mu\gamma_5 - \frac{i}{2}\gamma_\mu(\varphi + \gamma_5\pi)\right]\eta \\
\frac{1}{m}\delta\rho &= -\frac{1}{\sqrt{2}}(\varphi + \gamma_5\pi)\eta \\
\frac{1}{m}\delta\chi &= [i\hat{A} + i\hat{B}\gamma_5 + \varphi + \gamma_5\pi]\eta
\end{aligned} \tag{2.5}$$

It is easy to check that with the resulting fermionic mass terms:

$$\frac{1}{m}\mathcal{L}_m = \frac{1}{2}\bar\Psi_\mu\sigma^{\mu\nu}\Psi_\nu - \frac{i}{\sqrt{2}}(\bar\Psi\gamma)\rho + i(\bar\Psi\gamma)\chi - \sqrt{2}\bar\rho\chi + \frac{1}{2}\bar\chi\chi$$

the total Lagrangian is invariant (besides global supertransformations) under the following local gauge transformations:

$$\delta\Psi_\mu = \partial_\mu\xi - \frac{im}{2}\gamma_\mu\xi, \qquad \delta\rho = -\frac{m}{\sqrt{2}}\xi, \qquad \delta\chi = m\xi \tag{2.6}$$

From the last formula one can see which combination of two spinor fields plays the role of Goldstone one. Indeed, if we introduce two orthogonal combinations:

$$\tilde\rho = -\frac{1}{\sqrt{3}}\rho + \sqrt{\frac{2}{3}}\chi, \qquad \tilde\chi = \sqrt{\frac{2}{3}}\rho + \frac{1}{\sqrt{3}}\chi$$

then the fermionic mass terms take the form:

$$\frac{1}{m}\mathcal{L}_m = \frac{1}{2}\bar\Psi_\mu\sigma^{\mu\nu}\Psi_\nu + i\sqrt{\frac{3}{2}}(\bar\Psi\gamma)\tilde\rho + \bar{\tilde\rho}\tilde\rho - \frac{1}{2}\bar{\tilde\chi}\tilde\chi$$

which explicitly shows that we have spin 3/2 and spin 1/2 particles with equal masses. Moreover, by using this local gauge transformation with $\xi = -(\varphi + \gamma_5\pi)\eta$ and introducing gauge invariant derivatives for the scalar fields:

$$\nabla_\mu\varphi = \partial_\mu\varphi - mA_\mu, \qquad \nabla_\mu\pi = \partial_\mu\pi - mB_\mu$$

one can bring supertransformations for the fermions into the following simple form:

$$\begin{aligned}
\delta\Psi_\mu &= \left[-\frac{i}{4}\sigma^{\alpha\beta}(A_{\alpha\beta} - B_{\alpha\beta}\gamma_5)\gamma_\mu - \nabla_\mu(\varphi + \gamma_5\pi)\right]\eta \\
\delta\rho &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}[A_{\alpha\beta} + B_{\alpha\beta}\gamma_5]\eta \qquad \delta\chi = -i\hat\nabla(\varphi + \gamma_5\pi)\eta
\end{aligned} \tag{2.7}$$

Note here that we work with Majorana fermions (and Majorana representation of γ-matrices). In this, the γ_5 matrix plays the role of imaginary unit i. Then we can further simplify formula given above by introducing complex objects $C_\mu = (A_\mu + \gamma_5 B_\mu)$ and $z = \varphi + \gamma_5\pi$:

$$\begin{aligned}
\delta\Psi_\mu &= \left[-\frac{i}{4}\sigma^{\alpha\beta}\bar C_{\alpha\beta}\gamma_\mu - \nabla_\mu z\right]\eta \\
\delta\rho &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta}\eta \qquad \delta\chi = -i\hat\nabla z\eta
\end{aligned} \tag{2.8}$$

Now it is evident that we have one more symmetry — axial $\mathrm{U}(1)_A$ global symmetry, the axial charges for all fields being:

field	η	Ψ_μ, ρ, χ	C_μ, z
q_A	+1	0	–1

Thus, we have seen that it is important for construction of this supermultiplet to have possibility of making dual rotation of vector fields mixing massless supermultiplets. Also, there is a tight connection between vector fields (Higgs effect) and spin 3/2 (super-Higgs effect) masses. And indeed, the existence of dual versions of $N = 2$ supergravities and appropriate gaugings makes partial super-Higgs effect possible [16, 32–35].

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3. Massive spin 3/2 in $(A)dS_4$

In the (Anti) de Sitter space-time it is the (Anti) de Sitter group that plays the role of global background symmetry instead of Poincare group in Minkowski space. As a result, there is no strict definition of what is mass in such space. And indeed, a lot of controversy on this subject exists in the literature. The aim of this small section is to explain the definition of mass we (personally) adhere to using massive spin 3/2 particle as an example.

Anti de Sitter space is the constant curvature space without torsion or non-metricity, so the main difference from the Minkowski space is the replacement of ordinary partial derivatives by the covariant ones. We will use the following normalization here:

$$[\nabla_\mu, \nabla_\nu] = \frac{\kappa}{2}\sigma_{\mu\nu}, \qquad \kappa = -\frac{2\Lambda}{(d-1)(d-2)} = -\frac{\Lambda}{3} \tag{3.1}$$

where Λ — cosmological term. Now let us consider the quadratic Lagrangian for spin 3/2 Ψ_μ and spin 1/2 χ fields with the most general mass terms:

$$\mathcal{L} = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{\Psi}_\mu\gamma_5\gamma_\nu\nabla_\alpha\Psi_\beta + \frac{i}{2}\bar{\chi}\hat{\nabla}\chi + \frac{M}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + ia_1(\bar{\Psi}\gamma)\chi + \frac{a_2}{2}\bar{\chi}\chi \tag{3.2}$$

and require that it will be invariant under the following local gauge transformations:

$$\delta\Psi_\mu = \nabla_\mu\xi + i\alpha_1\gamma_\mu\xi \qquad \delta\chi = \alpha_2\xi$$

Simple calculations immediately give:

$$a_1 = \alpha_2, \qquad a_2 = 2M, \qquad \alpha_1 = -\frac{M}{2}, \qquad M^2 = \frac{2}{3}{\alpha_2}^2 + k$$

For the gauge invariant description of massive particles it is natural to define massless limit as the limit when Goldstone field(s) completely decouples from the the main gauge field. In the case at hands this means that it is the parameter a_1 determines the mass $a_1 \sim m$. As for the concrete normalization we will require that in the flat space limit our definition coincides with the usual one. Thus $a_1 = \sqrt{\frac{3}{2}}m$ and $M = \sqrt{m^2+\kappa}$. One of the peculiar features of (Anti) de Sitter spaces is the existence of so called forbidden mass regions [19−21]. And we see that different choices of what one call mass (M or m in the case considered) lead to drastically different physical interpretations, as we illustrate by the following simple picture:

For the fermionic fields with higher spins similar results can be easily obtained from that of [10]. Also note the paper [36] where group-theoretical arguments in favor of de Sitter space were given. At the same time for the bosonic particles our definition agrees perfectly with the one used by authors of [19−21].

4. $N = 1$ supermultiplet in AdS_4

In this section we consider the same massive $N = 1$ supermultiplet in Anti de Sitter space [22]. Now, besides the replacement of ordinary partial derivatives by the covariant

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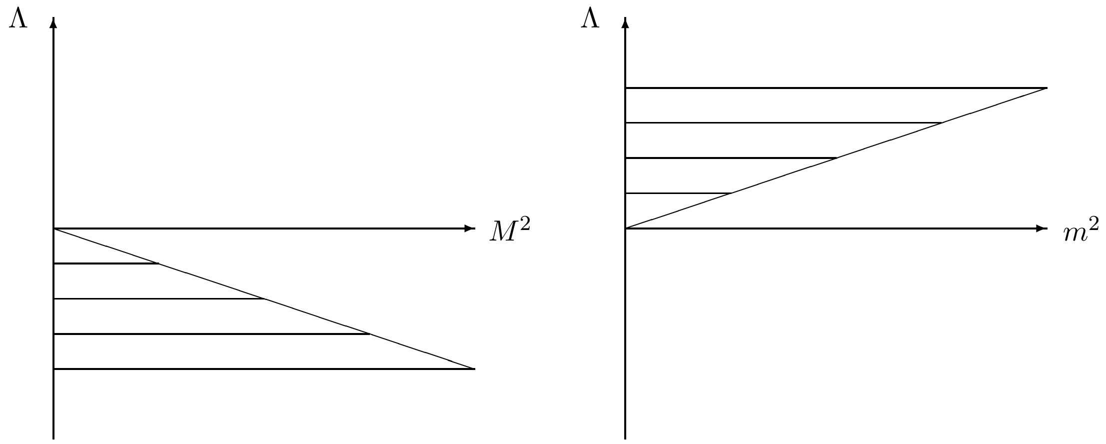

Figure 1: Forbidden regions.

ones, one has to take care on the definition of global supertransformations. The simple and natural choice (e.g. [37]) is to use the spinor η satisfying the relation:

$$\nabla_\mu \eta = -\frac{i\kappa_0}{2}\gamma_\mu \eta, \qquad {\kappa_0}^2 = \kappa$$

as a parameter of such "global" supertransformations.

Now we return back to the sum of kinetic terms for all fields where ordinary derivatives are replaced by covariant ones:

$$\mathcal{L}_0 = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{\Psi}_\mu \gamma_5 \gamma_\nu \nabla_\alpha \Psi_\beta + \frac{i}{2}\bar{\rho}\hat{\nabla}\rho + \frac{i}{2}\bar{\chi}\hat{\nabla}\chi - \frac{1}{4}{A_{\mu\nu}}^2 - \frac{1}{4}{B_{\mu\nu}}^2 + \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}(\partial_\mu\pi)^2 \quad (4.1)$$

In Anti de Sitter space this Lagrangian is no longer invariant under the initial supertransformations:

$$\delta_0\mathcal{L}_0 = -\frac{i\kappa_0}{\sqrt{2}}\bar{\Psi}_\mu[\cos(\theta)(A^{\mu\nu} - \gamma_5\tilde{A}^{\mu\nu}) + \sin(\theta)(\gamma_5 B^{\mu\nu} - \tilde{B}^{\mu\nu})]\gamma_\nu\eta + i\kappa_0\bar{\chi}\gamma^\mu(\partial_\mu\varphi - \gamma_5\partial_\mu\pi)\eta$$

We proceed by adding the most general mass terms for the fermions as well as one derivative terms for the bosons:

$$\begin{aligned}\mathcal{L}_1 = {} & \frac{a_1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + ia_2(\bar{\Psi}\gamma)\rho + ia_3(\bar{\Psi}\gamma)\chi + a_4\bar{\rho}\rho + a_5\bar{\rho}\chi + \frac{a_6}{2}\bar{\chi}\chi - \\ & - m_1 A^\mu\partial_\mu\varphi - m_2 B^\mu\partial_\mu\pi \end{aligned} \quad (4.2)$$

and the most general additional terms for the fermionic supertransformations:

$$\begin{aligned}\delta_1\Psi_\mu &= [\alpha_1 A_\mu + \alpha_2 B_\mu\gamma_5 + i\alpha_3\gamma_\mu\varphi + i\alpha_4\gamma_\mu\gamma_5\pi]\eta \\ \delta_1\rho &= [i\beta_1\hat{A} + i\beta_2\hat{B}\gamma_5 + \beta_3\varphi + \beta_4\gamma_5\pi]\eta\end{aligned} \quad (4.3)$$

$$\delta_1\chi = [i\beta_5\hat{A} + i\beta_6\hat{B}\gamma_5 + \beta_7\varphi + \beta_8\gamma_5\pi]\eta$$

Requirement that all variations containing one derivative cancel gives:

$$\begin{aligned}
a_1 &= -M, \quad a_2 = \frac{M}{\sqrt{2}}\sin(2\theta), \quad \alpha_1 = -M\sqrt{2}\cos(\theta) \quad \alpha_2 = -M\sqrt{2}\sin(\theta) \\
a_3 &= m_1\sqrt{2}\cos(\theta) = m_2\sqrt{2}\sin(\theta), \qquad \alpha_3 = \alpha_4 = -\frac{a_3}{2} \\
a_4 &= \beta_1 = \beta_2 = 0, \qquad \beta_3 = a_5 + m_1\sin(\theta), \qquad \beta_4 = a_5 + m_2\cos(\theta) \\
a_5 &= -\frac{m_1}{\sin(\theta)} = -\frac{m_2}{\cos(\theta)}, \quad \beta_5 = m_1, \quad \beta_6 = m_2, \quad \beta_7 = a_6 - \kappa_0, \quad \beta_8 = a_6 + \kappa_0
\end{aligned}$$

Here $M = \frac{\kappa_0}{\cos(2\theta)}$. We see that it is the mixing angle θ (together with cosmological term) determines all masses in this case. Recall, that in flat space we have $\sin(\theta) = \cos(\theta) = \frac{1}{\sqrt{2}}$, while here it is the singular point. Indeed, the flat space results could be correctly reproduced only by taking simultaneous limits $\kappa_0 \to 0$ and $\theta \to \frac{\pi}{4}$ so that $\kappa_0\tan(2\theta)$ remains to be fixed.

At last we add appropriate mass terms for bosons:

$$\mathcal{L}_2 = \frac{{m_1}^2}{2}{A_\mu}^2 + \frac{{m_2}^2}{2}{B_\mu}^2 + b_1\varphi^2 + b_2\pi^2 \tag{4.4}$$

and require that all variations without derivatives cancel. This gives:

$$m_1 = M\sqrt{2}\sin(\theta), \qquad m_2 = M\sqrt{2}\cos(\theta), \qquad a_6 = -M, \qquad b_1 = b_2 = 0$$

The resulting mass terms for the fermions look like:

$$\mathcal{L}_m = -\frac{M}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + \frac{iM\sin(2\theta)}{\sqrt{2}}(\bar{\Psi}\gamma)\rho + iM\sin(2\theta)(\bar{\Psi}\gamma)\chi - M\sqrt{2}\bar{\rho}\chi - \frac{M}{2}\bar{\chi}\chi \tag{4.5}$$

In this, besides the supertransformations, Lagrangian is invariant under the following local gauge transformations:

$$\delta\Psi_\mu = \nabla_\mu\xi + \frac{iM}{2}\gamma_\mu\xi, \qquad \delta\rho = \frac{M\sin(2\theta)}{\sqrt{2}}\xi, \qquad \delta\chi = M\sin(2\theta)\xi \tag{4.6}$$

Comparing this formula with the results of previous sections, one can conclude that it is the combination $m = M\sin(2\theta) = \kappa_0\tan(2\theta)$ determines the mass for spin 3/2 particle. So we have four massive fields with masses (which become equal in the limit $\theta \to \pi/4$):

$$m_{3/2} = m, \quad m_1 = \frac{m}{\sqrt{2}\cos(\theta)}, \quad m_{1'} = \frac{m}{\sqrt{2}\sin(\theta)}, \quad m_{1/2} = \frac{m}{\sin(2\theta)} \tag{4.7}$$

As in the flat case, introducing gauge invariant derivatives for scalar fields:

$$\nabla_\mu\varphi = \partial_\mu\varphi - m_1A_\mu, \qquad \nabla_\mu\pi = \partial_\mu\pi - m_2B_\mu,$$

and making local gauge transformation with $\xi = (\cot(\theta)\varphi + \tan(\theta)\pi)\eta$ one can bring supertransformations for fermions into relatively simple form:

$$\begin{aligned}
\delta\Psi_\mu &= -\frac{i}{2\sqrt{2}}\sigma^{\alpha\beta}[\cos(\theta)A_{\alpha\beta} - \sin(\theta)B_{\alpha\beta}\gamma_5]\gamma_\mu\eta + (\cot(\theta)\nabla_\mu\varphi + \tan(\theta)\nabla_\mu\pi\gamma_5)\eta \\
\delta\rho &= -\frac{1}{2}\sigma^{\alpha\beta}[\sin(\theta)A_{\alpha\beta} + \cos(\theta)B_{\alpha\beta}\gamma_5]\eta \qquad \delta\chi = -i\hat{\nabla}(\varphi + \gamma_5\pi)\eta
\end{aligned} \tag{4.8}$$

Note, that in Anti de Sitter space there is no axial $\mathrm{U}(1)_A$ symmetry.

5. $S = 2$ supermultiplet in AdS_4

This paper devoted mainly to construction of massive spin 3/2 supermultiplets, but it is instructive to compare with the next to simplest case — massive spin 2 supermultiplet. In flat space such multiplets were constructed in [23] (see also [24, 38, 25]), so we consider AdS_4 case. Massive $N = 1$ spin 2 supermultiplet contains four massive fields $(2, 3/2, 3/2', 1)$. Taking into account that in the massless limit (in flat space, see below) massive spin 2 particle decompose into massless spin 2, spin 1 and spin 0 ones, we have to use four massless supermultiplets for our construction:

$$\begin{pmatrix} & 2 & \\ \frac{3}{2} & & \frac{3}{2}' \\ & 1 & \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{3}{2}' \\ 1 \end{pmatrix} \oplus \begin{pmatrix} 1' \\ \frac{1}{2} \end{pmatrix} \oplus \begin{pmatrix} \frac{1}{2}' \\ 0, 0' \end{pmatrix}$$

We denote appropriate fields as $(h_{\mu\nu}, \Psi_\mu)$, (Ω_μ, A_μ), (B_μ, ρ) and (χ, φ, π) and start with the sum of kinetic terms for all fields (with ordinary partial derivatives replaced by covariant ones):

$$\begin{aligned} \mathcal{L}_0 = {} & \frac{1}{2}\nabla^\mu h^{\alpha\beta}\nabla_\mu h_{\alpha\beta} - (\nabla h)^\mu(\nabla h)_\mu + (\nabla h)^\mu \nabla_\mu h - \frac{1}{2}\nabla^\mu h \nabla_\mu h - \\ & - \frac{1}{4}{A_{\mu\nu}}^2 + \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{1}{4}{B_{\mu\nu}}^2 + \frac{1}{2}(\partial_\mu\pi)^2 + \\ & + \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\bar\Psi_\mu\gamma_5\gamma_\nu\nabla_\alpha\Psi_\beta + \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\bar\Omega_\mu\gamma_5\gamma_\nu\nabla_\alpha\Omega_\beta + \frac{i}{2}\bar\rho\hat\nabla\rho + \frac{i}{2}\bar\chi\hat\nabla\chi \end{aligned} \tag{5.1}$$

It is crucial for the whole construction that we again have one vector and one axial-vector fields and the possibility to make dual mixing of two supermultiplets containing these fields. So we will use the following ansatz for supertransformations:

$$\begin{aligned} \delta_0 h_{\mu\nu} &= i(\bar\Psi_{(\mu}\gamma_{\nu)}\eta) \qquad \delta_0\Psi_\mu = -\sigma^{\alpha\beta}\nabla_\alpha h_{\beta\mu} \\ \delta_0\Omega_\mu &= -\frac{i}{2\sqrt{2}}\sigma^{\alpha\beta}(\cos(\theta)A_{\alpha\beta} - \sin(\theta)B_{\alpha\beta}\gamma_5)\gamma_\mu\eta \\ \delta_0 A_\mu &= \sqrt{2}\cos(\theta)(\bar\Omega_\mu\eta) + i\sin(\theta)(\bar\rho\gamma_\mu\eta) \\ \delta_0 B_\mu &= \sqrt{2}\sin(\theta)(\bar\Omega_\mu\gamma_5\eta) + i\cos(\theta)(\bar\rho\gamma_\mu\gamma_5\eta) \\ \delta_0\rho &= -\frac{1}{2}\sigma^{\alpha\beta}(\sin(\theta)A_{\alpha\beta} + \cos(\theta)B_{\alpha\beta}\gamma_5)\eta \\ \delta_0\chi &= -i\gamma_\mu(\partial_\mu\varphi + \partial_\mu\pi\gamma_5)\eta \qquad \delta_0\varphi = (\bar\chi\eta) \qquad \delta_0\pi = (\bar\chi\gamma_5\eta) \end{aligned} \tag{5.2}$$

In AdS space the sum of kinetic terms is not invariant under these transformations any more and we must take it into account in the subsequent calculations. The next question is which fields play the role of Goldstone ones making gauge fields massive. The choice for bosonic fields is unambiguous — vector A_μ and scalar φ fields for $h_{\mu\nu}$ and pseudo-scalar π for B_μ one. But for the fermions situation is more complicated. Recall that in AdS case we have no axial $\mathrm{U}(1)_A$ symmetry which could restrict possible choice, thus we have to consider the most general mass terms for the fermions. So we add to our Lagrangian:

$$\begin{aligned}
\mathcal{L}_1 &= m\sqrt{2}(h^{\mu\nu}\nabla_\mu A_\nu - h(\nabla A)) - M\sqrt{3}A^\mu\partial_\mu\varphi - \tilde{m}B^\mu\partial_\mu\pi - \\
&\quad -\frac{a_1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu - a_2\bar{\Psi}_\mu\sigma^{\mu\nu}\Omega_\nu - \frac{a_3}{2}\bar{\Omega}_\mu\sigma^{\mu\nu}\Omega_\nu + ia_4(\bar{\Psi}\gamma)\rho + ia_5(\bar{\Psi}\gamma)\chi + \\
&\quad +ia_6(\bar{\Omega}\gamma)\rho + ia_7(\bar{\Omega}\gamma)\chi + \frac{a_8}{2}\bar{\rho}\rho + a_9\bar{\rho}\chi + \frac{a_{10}}{2}\bar{\chi}\chi
\end{aligned} \tag{5.3}$$

where $M = \sqrt{m^2 + 2\kappa}$, and require that all variations with one derivative cancel (making necessary corrections for fermionic supertransformations). This gives us:

$$\begin{array}{lllll}
\sin(\theta) = \dfrac{\sqrt{3}}{2}, & \cos(\theta) = \dfrac{1}{2}, & \tilde{m} = M & & \\
a_1 = \kappa_0, & a_2 = m, & a_3 = -2\kappa_0, & a_4 = m\sqrt{\dfrac{3}{2}}, & a_5 = 0 \\
a_6 = -\sqrt{\dfrac{3}{2}}\kappa_0, & a_7 = \sqrt{\dfrac{3}{2}}M, & a_8 = 0, & a_9 = -2M &
\end{array}$$

Note that in sharp contrast with the massive spin 3/2 case now the mixing angle θ is fixed (and has the same value as in flat case) so all masses are determined by spin 2 mass m and cosmological constant κ. We proceed by adding appropriate mass terms for bosonic fields:

$$\mathcal{L}_2 = -\frac{m^2 - 2\kappa}{2}h^{\mu\nu}h_{\mu\nu} + \frac{m^2 + \kappa}{2}h^2 - \sqrt{\frac{3}{2}}mMh\varphi + 3\kappa {A_\mu}^2 + m^2\varphi^2 + \frac{M^2}{2}{B_\mu}^2 \tag{5.4}$$

and requiring cancellation of all variations without derivatives. This fixes the last unknown parameter $a_{10} = 2\kappa_0$ and the structure of additional terms in fermionic supertransformations:

$$\begin{aligned}
\delta_1\Psi_\mu &= \left[i\kappa_0 h_{\mu\nu}\gamma^\nu - \frac{m}{\sqrt{2}}\gamma_\mu\hat{A} - m\sqrt{\frac{3}{2}}\gamma_5 B_\mu\right]\eta \\
\delta_1\Omega_\mu &= \left[imh_{\mu\nu}\gamma^\nu + \kappa_0\sqrt{2}A_\mu + \kappa_0\sqrt{6}\gamma_5 B_\mu - \frac{i}{2}\sqrt{\frac{3}{2}}M\gamma_\mu(\varphi + \gamma_5\pi)\right]\eta \\
\delta_1\rho &= \left[-\frac{M}{2}\varphi - \frac{3M}{2}\gamma_5\pi\right]\eta \\
\delta_1\chi &= [iM\sqrt{3}\hat{A} + iM\hat{B}\gamma_5 + \kappa_0\varphi + 3\kappa_0\gamma_5\pi]\eta
\end{aligned} \tag{5.5}$$

Recall that in the flat case [23] due to axial $\mathrm{U}(1)_A$ symmetry fermionic mass terms were the Dirac ones. For the non-zero cosmological term the structure of these terms become more complicated:

$$\begin{aligned}
\mathcal{L}_m &= -\frac{\kappa_0}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu - m\bar{\Psi}_\mu\sigma^{\mu\nu}\Omega_\nu - \kappa_0\bar{\Omega}_\mu\sigma^{\mu\nu}\Omega_\nu + im\sqrt{\frac{3}{2}}(\bar{\Psi}\gamma)\rho - \\
&\quad -i\kappa_0\sqrt{\frac{3}{2}}(\bar{\Omega}\gamma)\rho + iM\sqrt{\frac{3}{2}}(\bar{\Omega}\gamma)\chi - 2M\bar{\rho}\chi + \kappa_0\bar{\chi}\chi
\end{aligned} \tag{5.6}$$

Nevertheless, it is not hard to check that the Lagrangian obtained is invariant (besides supertransformations) under two local gauge transformations:

$$\begin{array}{llll}
\delta\Psi_\mu = \nabla_\mu\xi_1 + \dfrac{i\kappa_0}{2}\gamma_\mu\xi_1, & \delta\Omega_\mu = \dfrac{im}{2}\gamma_\mu\xi_1, & \delta\rho = m\sqrt{\dfrac{3}{2}}\xi_1 & \\
\delta\Psi_\mu = \dfrac{im}{2}\gamma_\mu\xi_2, & \delta\Omega_\mu = \nabla_\mu\xi_2 - i\kappa_0\gamma_\mu\xi_2, & \delta\rho = -\kappa_0\sqrt{\dfrac{3}{2}}\xi_2, & \delta\chi = M\sqrt{\dfrac{3}{2}}\xi_2
\end{array} \tag{5.7}$$

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As is known [3, 6], in the (A)dS space massive spin s particle decompose in the massless limit into massless spin s and massive spin s-1 ones. Similarly, in the massless limit $m \to 0$ our massive spin 2 supermultiplets decompose into massless $(2, 3/2)$ supermultiplet and massive $(3/2, 1, 1', 1/2)$ one with mass $M = 2\kappa_0$ and mixing angle $\sin(2\theta) = \frac{\sqrt{3}}{2}$. Note that in the paper [22] this corresponds to the value $\varepsilon = 1/2$.

All the formulas could be greatly simplified if we introduce gauge invariant derivatives:

$$D_\mu h_{\alpha\beta} = \nabla_\mu h_{\alpha\beta} - \frac{m}{\sqrt{2}} A_\mu g_{\alpha\beta}, \qquad D_\mu \varphi = \partial_\mu \varphi - M\sqrt{3} A_\mu, \qquad D_\mu \pi = \partial_\mu \pi - M B_\mu \tag{5.8}$$

as well as notation $H_{\mu\nu} = h_{\mu\nu} - \frac{m}{M\sqrt{6}} \varphi g_{\mu\nu}$ and make two local gauge transformations with the parameters:

$$\xi_1 = \frac{m}{M\sqrt{6}}(\varphi + 3\pi\gamma_5)\eta \qquad \xi_2 = -\frac{\kappa_0\sqrt{6}}{M}\left(\frac{1}{3}\varphi + \pi\gamma_5\right)\eta$$

Then the resulting supertransformations for the fermions take the form:

$$\begin{aligned}
\delta\Psi_\mu &= \left[-\sigma^{\alpha\beta} D_\alpha h_{\beta\mu} + \frac{m}{M\sqrt{6}}(D_\mu\varphi + 3D_\mu\pi\gamma_5) + i\kappa_0 H_{\mu\nu}\gamma^\nu \right]\eta \\
\delta\Omega_\mu &= \left[-\frac{i}{4\sqrt{2}}\sigma^{\alpha\beta}(A_{\alpha\beta} - \sqrt{3}B_{\alpha\beta}\gamma_5)\gamma_\mu - \frac{\kappa_0\sqrt{6}}{M}\left(\frac{1}{3}D_\mu\varphi + D_\mu\pi\gamma_5\right) + imH_{\mu\nu}\gamma^\nu \right]\eta \\
\delta\rho &= -\frac{1}{4}\sigma^{\alpha\beta}(\sqrt{3}A_{\alpha\beta} + B_{\alpha\beta}\gamma_5)\eta \qquad \delta\chi = -i\gamma_\mu(D_\mu\varphi + D_\mu\pi\gamma_5)\eta
\end{aligned} \tag{5.9}$$

It is interesting that the structure of terms containing scalar fields gives us one more example of flat space limit — massless limit ambiguity well known for the massive spin 2 [26–28] and spin 3/2 [29–31] particles. Indeed, if one takes massless limit keeping cosmological term fixed, one gets:

$$\delta\Psi_\mu \sim 0, \qquad \delta\Omega_\mu \sim -\sqrt{3}\left(\frac{1}{3}D_\mu\varphi + D_\mu\pi\gamma_5\right)\eta$$

At the same time, in the flat space limit with fixed m we get:

$$\delta\Psi_\mu \sim \frac{1}{\sqrt{6}}(D_\mu\varphi + 3D_\mu\pi\gamma_5)\eta \qquad \delta\Omega_\mu \sim 0$$

6. $N = 2$ supermultiplet

Now we return back to flat Minkowski space and consider massive spin 3/2 supermultiplets with extended supersymmetries. Our next example — massive $N = 2$ supermultiplet containing one spin 3/2, four spin 1, six spin 1/2 and four spin 0 particles. Simple calculations show that in the massless limit we obtain one spin 3/2 supermultiplet, doublet of vector supermultiplets and one hypermultiplet:

$$\begin{pmatrix} \frac{3}{2} \\ 4 \otimes 1 \\ 6 \otimes \frac{1}{2} \\ 4 \otimes 0 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} \frac{3}{2} \\ 2 \otimes 1 \\ \frac{1}{2} \end{pmatrix} \oplus 2 \otimes \begin{pmatrix} 1 \\ 2 \otimes \frac{1}{2} \\ 2 \otimes 0 \end{pmatrix} \oplus \begin{pmatrix} 2 \otimes \frac{1}{2} \\ 4 \otimes 0 \end{pmatrix}$$

We denote all these fields as $(\Psi_\mu, A_\mu{}^i, \rho)$, $(B_\mu{}^i, \Omega_i{}^j, z_i)$ and (χ, λ, Φ_i) and start with the sum of their kinetic terms:

$$\mathcal{L}_0 = \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\bar{\Psi}_\mu\gamma_5\gamma_\nu\partial_\alpha\Psi_\beta - \frac{1}{4}A_{\mu\nu}{}^2 - \frac{1}{4}B_{\mu\nu}{}^2 + \frac{i}{2}\bar{\rho}\hat{\partial}\rho + \frac{i}{2}\bar{\Omega}_i{}^j\hat{\partial}\Omega_i{}^j + \\ + \frac{i}{2}\bar{\chi}\hat{\partial}\chi + \frac{i}{2}\bar{\lambda}\hat{\partial}\lambda + \frac{1}{2}\partial_\mu\bar{z}^i\partial_\mu z_i + \frac{1}{2}\partial_\mu\bar{\Phi}^i\partial_\mu\Phi_i \quad (6.1)$$

It is important that the massive supermultiplet we are going to construct must have total $\mathrm{U}(2) = \mathrm{SU}(2) \otimes \mathrm{U}(1)_A$ symmetry. It is again crucial that we have doublet of vector and doublet of axial-vector fields in our disposal. This allows us by making dual transformation mixing two vector supermultiplets introduce complex objects $C_\mu{}^i = A_\mu{}^i + \gamma_5 B_\mu{}^i$. Also, this $\mathrm{U}(2)$ symmetry dictates our choice of parametrisation for hypermultiplet (there exists three different ones). Thus we take the following form of supertransformations for massless supermultiplets:

$$\begin{aligned}
\delta\Psi_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta}{}^i\eta_i & \delta\rho &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta i}\varepsilon^{ij}\eta_j \\
\delta\bar{C}_{\mu i} &= 2(\bar{\Psi}_\mu\eta_i) + i\sqrt{2}(\bar{\Omega}_j{}^i\gamma_\mu\eta_j) & \delta C_\mu{}^i &= i\sqrt{2}(\bar{\rho}\gamma_\mu\varepsilon^{ij}\eta_j) \qquad (6.2) \\
\delta\Omega_i{}^j &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta}{}^j\eta_i - i\varepsilon_{ik}\hat{\partial}z_j\eta^k & \delta\bar{z}^i &= 2\varepsilon^{jk}(\bar{\Omega}_j{}^i\eta_k) \\
\delta\chi &= i\hat{\partial}\varepsilon^{ij}\Phi_i\eta_j & \delta\lambda &= -i\hat{\partial}\bar{\Phi}^i\eta_i \\
\delta\bar{\Phi}^i &= -2\varepsilon^{ij}(\bar{\chi}\eta_j) & \delta\Phi_i &= 2(\bar{\lambda}\eta_i) \qquad (6.3)
\end{aligned}$$

In the complex notations the SU(2) symmetry of our construction is explicit, while the axial $\mathrm{U}(1)_A$ symmetry is achieved by the following assignment of axial charges:

field	ρ	η_i	Ψ_μ, $\Omega_i{}^j$, χ	$C_\mu{}^i$, z_i, Φ_i	λ
q_A	+2	+1	0	−1	−2

This axial $\mathrm{U}(1)_A$ symmetry restricts possible form of fermionic mass terms and the most general terms compatible with it look like:

$$\frac{1}{m}\mathcal{L}_1 = -\frac{1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + ia_1(\bar{\Psi}\gamma)\Omega + ia_2(\bar{\Psi}\gamma)\chi + a_3\bar{\Omega}_i{}^j\Omega_j{}^i + a_4\bar{\Omega}\Omega + a_5\bar{\Omega}\chi + a_6\bar{\rho}\lambda \quad (6.4)$$

As for the vector fields, we have two complex doublet of scalars which both could play the role of Goldstone fields. Straightforward calculations show that it is the combination $z_i + \Phi_i$ that have to be eaten by vector fields, leaving other combination $z_i - \Phi_i$ as physical massive scalar fields. So we get mass terms for bosonic fields:

$$\mathcal{L}_2 = \frac{m}{2\sqrt{2}}[\varepsilon_{ij}C_\mu{}^i\partial_\mu(\bar{z}^j + \bar{\Phi}^j) + (h.c.)] + \frac{m^2}{2}C_\mu{}^i\bar{C}_{\mu i} - \frac{m^2}{4}|z_i - \Phi_i|^2 \quad (6.5)$$

determine the coefficients for fermionic mass terms:

$$a_1 = -a_2 = \frac{1}{\sqrt{2}}, \qquad a_3 = -a_4 = \frac{1}{2}, \qquad a_5 = a_6 = 1$$

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as well as structure of additional terms for fermionic supertransformations:

$$\begin{aligned}
\frac{1}{m}\delta_1\Psi_\mu &= -C_\mu{}^i\eta_i - \frac{i}{2\sqrt{2}}\gamma_\mu(z_i+\Phi_i)\varepsilon^{ij}\eta_j \qquad \frac{1}{m}\delta_1\rho = -\frac{1}{2}(\bar{z}^i - \bar{\Phi}^i)\eta_i \\
\frac{1}{m}\delta_1\Omega_i{}^j &= -\frac{i}{\sqrt{2}}\gamma^\mu[C_\mu{}^i\eta_j - \delta_j{}^i C_\mu{}^k\eta_k] + \frac{1}{2}\varepsilon^{jk}(z_k-\Phi_k)\eta_i - \delta_i{}^j\Phi_k\varepsilon^{kl}\eta_l \\
\frac{1}{m}\delta_1\chi &= -\frac{i}{\sqrt{2}}\gamma^\mu C_\mu{}^i\eta_i + z_i\varepsilon^{ij}\eta_j \qquad\qquad \frac{1}{m}\delta_1\lambda = -\frac{i}{\sqrt{2}}\gamma^\mu\bar{C}_{\mu i}\varepsilon^{ij}\eta_j
\end{aligned} \tag{6.6}$$

It is hardly comes as a surprise that besides global supersymmetry and U(2) symmetry our Lagrangian is invariant under the local gauge transformations:

$$\delta\Psi_\mu = \partial_\mu\xi + \frac{im}{2}\gamma_\mu\xi, \qquad \delta\Omega_i{}^j = \frac{m}{\sqrt{2}}\delta_i{}^j\xi, \qquad \delta\chi = -\frac{m}{\sqrt{2}}\xi \tag{6.7}$$

Making such transformation with: $\xi = \frac{1}{\sqrt{2}}(z_i+\Phi_i)\varepsilon^{ij}\eta_j$ and introducing gauge invariant derivatives for scalar fields:

$$D_\mu z_i = \partial_\mu z_i - \frac{m}{\sqrt{2}}\varepsilon_{ij}C_\mu{}^j, \qquad D_\mu\Phi_i = \partial_\mu\Phi_i - \frac{m}{\sqrt{2}}\varepsilon_{ij}C_\mu{}^j$$

we obtain the final form of fermionic supertransformations:

$$\begin{aligned}
\delta\Psi_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta}{}^i\eta_i + \frac{1}{\sqrt{2}}D_\mu(z_i+\Phi_i)\varepsilon^{ij}\eta_j \\
\delta\rho &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta i}\varepsilon^{ij}\eta_j - \frac{m}{2}(\bar{z}^i - \bar{\Phi}^i)\eta_i \\
\delta\Omega_i{}^j &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta}{}^j\eta_i - i\varepsilon_{ik}\hat{D}z_j\eta^k + +\frac{m}{2}\varepsilon^{jk}(z_k-\Phi_k)\eta_i + \frac{m}{2}\delta_i{}^j(z_k-\Phi_k)\varepsilon^{kl}\eta_l \\
\delta\chi &= i\hat{D}\Phi_i\varepsilon^{ij}\eta_j + \frac{m}{2}(z_i-\Phi_i)\varepsilon^{ij}\eta_j \qquad \delta\lambda = -i\hat{D}\bar{\Phi}^i\eta_i
\end{aligned} \tag{6.8}$$

Such supermultiplet has to appear when $N = 3$ or $N = 4$ supergravity is spontaneously broken up to $N = 2$ and indeed such breaking turns out to be possible as was shown in [39–42] (see also [43]).

7. $N = 3$ supermultiplet

Our next example is massive $N = 3$ supermultiplet containing one spin 3/2, six spin 1, fourteen spin 1/2 and fourteen spin 0 particles. It easy to check that in the massless limit we will get one spin 3/2 supermultiplet $(3/2, 3\otimes 1, 3\otimes 1/2, 2\otimes 0)$ and three vector supermultiplets:

$$\begin{pmatrix} \frac{3}{2} \\ 6\otimes 1 \\ 14\otimes\frac{1}{2} \\ 14\otimes 0 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{3}{2} \\ 3\otimes 1 \\ 3\otimes\frac{1}{2} \\ 2\otimes 0 \end{pmatrix} \oplus 3\otimes \begin{pmatrix} 1 \\ 4\otimes\frac{1}{2} \\ 6\otimes 0 \end{pmatrix}$$

Really, this case is very similar to the previous one (and even more simple due to the absence of hypermultiplet). Again it is crucial that we have two triplets of (axial-)vector fields so we can arrange them into one comples triplet. As a result we get SU(3) invariant supertransformations leaving the sum of kinetic terms invariant:

$$\begin{aligned}
\delta_0\Psi_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta}{}^i\eta_i \\
\delta_0\bar{C}_\mu{}^i &= 2(\bar{\Psi}_\mu\eta_i) + i\sqrt{2}(\bar{\rho}_j{}^i\gamma_\mu\eta_j) \qquad\qquad \delta_0 C_{\mu i} = -i\sqrt{2}\varepsilon^{ijk}(\bar{\chi}_j\gamma_\mu\eta_k) \\
\delta_0\chi^i &= -\frac{1}{2\sqrt{2}}\varepsilon^{ijk}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta j}\eta_k - i\hat{\partial}z\eta_i \qquad\qquad \delta_0\bar{z} = 2(\bar{\chi}^i\eta_i) \\
\delta_0\rho_i{}^j &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta}{}^j\eta_i - i\hat{\partial}\varepsilon^{ikl}\Phi_{jk}\eta_l \qquad \delta_0\lambda_i = -i\hat{\partial}\bar{\Phi}^{ij}\eta_j \\
\delta_0\bar{\Phi}^{ij} &= 2(\bar{\rho}_k{}^i\varepsilon^{kjl}\eta_l) \qquad\qquad \delta_0\Phi_{ij} = 2(\bar{\lambda}_i\eta_j)
\end{aligned} \tag{7.1}$$

Moreover, with the appropriate assignment of axial charges:

field	η_i	Ψ_μ, $\rho_i{}^j$	χ^i	λ_i	$C_\mu{}^i$, Φ_{ij}	z
q_A	+1	0	+2	–2	–1	–3

we gain $\mathrm{U}(1)_A$ invariance as well. Among scalar fields there is only one candidate for the role of Goldstone field, namely antisymmetric part of $\Phi_{[ij]}$ leaving symmetric part as physical massive scalars. So the mass terms for bosons look like:

$$\mathcal{L}_b = -\frac{m}{2\sqrt{2}}[\varepsilon^{ijk}\bar{C}_{\mu i}\partial_\mu\Phi_{jk} + h.c.] + \frac{m^2}{2}[(A_\mu{}^i)^2 + (B_\mu{}^i)^2 - \bar{z}z - \bar{\Phi}^{(ij)}\Phi_{(ij)}] \tag{7.2}$$

while the most general fermionic mass terms compatible with U(3) invariance have the form:

$$\frac{1}{m}\mathcal{L}_f = -\frac{1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + ia_1(\bar{\Psi}\gamma)\rho_i{}^i + \frac{a_2}{2}\bar{\rho}_i{}^j\rho_j{}^i + \frac{a_3}{2}\bar{\rho}\rho + a_4\bar{\chi}^i\lambda_i \tag{7.3}$$

Then the requirement that the whole Lagrangian be supersymmetric fixes the unknown coefficients:

$$a_1 = \frac{1}{\sqrt{2}}, \qquad a_2 = 1, \qquad a_3 = -1, \qquad a_4 = -1$$

which lead to invariance of the Lagrangian under the local gauge transformations:

$$\delta\Psi_\mu = \partial_\mu\xi + \frac{im}{2}\gamma_\mu\xi \qquad \delta\rho_i{}^j = \frac{m}{\sqrt{2}}\delta_i{}^j\xi$$

and also fixes the structure of fermionic supertransformations. By using local gauge invariance and introducing gauge covariant derivatives, supertransformations for fermions could be casted to the form:

$$\begin{aligned}
\delta\Psi_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta}{}^i\eta_i + \frac{i}{\sqrt{2}}\varepsilon^{ijk}D_\mu z_{ij}\eta_k \\
\delta\chi^i &= -\frac{1}{2\sqrt{2}}\varepsilon^{ijk}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta j}\eta_k - i\hat{\partial}z\eta_i + m\bar{z}^{(ij)}\eta_j \\
\delta\rho_i{}^j &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta}{}^j\eta_i - i\gamma^\mu\varepsilon^{ikl}(\partial_\mu z_{(jk)} + D_\mu z_{[jk]})\eta_l + m\varepsilon^{jkl}z_{(ik)}\eta_l \\
\delta\lambda_i &= -i\gamma^\mu(\partial_\mu\bar{z}^{(ij)} + D_\mu\bar{z}^{[ij]})\eta_j + mz\eta_i
\end{aligned} \tag{7.4}$$

Such massive supermultiplet really appears then $N = 4$ supergravity is broken up to $N = 3$ [40, 42, 43]. Note that there is an interesting and still open question on the so called shadow supermultiplets that appear in some compactifications [44]. It would be interesting to investigate possible interactions of such massive $N = 3$ supermultiplets with $N = 3$ supergravity (without any other supermultiplets).

8. $N = 2$ supermultiplet with central charge

As is well known any massive supermultiplet without central charges having $N \geq 4$ supersymmetries necessarily contains particles with spin greater than 3/2. So we turn to the massive supermultiplets with central charges and start with the simplest example — with $N = 2$ supersymmetry. This multiplet contains two equal sets of particles corresponding to that of massive spin 3/2 supermultiplet with $N = 1$ supersymmetry, so the counting of fields in the massless limit is the same as before. But now we have to arrange all fields into massless $N = 2$ supermultiplets:

$$2 \otimes \begin{pmatrix} \frac{3}{2} \\ 2 \otimes 1 \\ \frac{1}{2} \end{pmatrix} \quad \Rightarrow \quad 2 \otimes \begin{pmatrix} \frac{3}{2} \\ 2 \otimes 1 \\ \frac{1}{2} \end{pmatrix} \oplus \begin{pmatrix} 2 \otimes \frac{1}{2} \\ 4 \otimes 0 \end{pmatrix}$$

For the hypermultiplet we will use the same parametrisation as before:

$$\begin{aligned} \delta\chi &= -i\varepsilon^{ij}\hat{\partial} z_i \eta_j \qquad & \delta\bar{z}^i &= 2\varepsilon^{ij}(\bar{\chi}\eta_j) \\ \delta\psi &= -i\hat{\partial}\bar{z}^i \eta_i \qquad & \delta z_i &= 2(\bar{\psi}\eta_i) \end{aligned} \tag{8.1}$$

As for the spin 3/2 supermultiplets, the main trick is again to use dual transformation for vector fields so that they enter through the complex combinations only. Indeed, it is not hard to check that sum of the kinetic terms for all fields is invariant under the following global $N = 2$ supertransformations:

$$\begin{aligned} \delta\Psi_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta i}\varepsilon^{ij}\eta^j \qquad & \delta\Omega_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu \bar{C}_{\alpha\beta}{}^i \eta_i \\ \delta\bar{C}_\mu{}^i &= 2\varepsilon^{ij}(\bar{\Psi}_\mu\eta_j) + i\sqrt{2}\varepsilon^{ij}(\bar{\lambda}\gamma_\mu\eta_j) \qquad & \delta C_{\mu i} &= 2(\bar{\Omega}_\mu\eta_i) + i\sqrt{2}(\bar{\rho}\gamma_\mu\eta_i) \\ \delta\lambda &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta i}\varepsilon^{ij}\eta_j \qquad & \delta\rho &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta}{}^i\eta_i \end{aligned} \tag{8.2}$$

The choice for the bosonic mass terms is unique:

$$\mathcal{L}_b = -\frac{m}{2}(\bar{C}_\mu{}^i\partial_\mu z_i + h.c.) + \frac{m^2}{2}\bar{C}_\mu{}^i C_{\mu i} \tag{8.3}$$

This time we have only $\mathrm{SU}(2) \simeq USp(2)$ global symmetry and no axial $\mathrm{U}(1)_A$ one so we have to consider the most general fermionic mass terms:

$$\begin{aligned} \frac{1}{m}\mathcal{L}_f = & -\frac{1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + ia_1(\bar{\Psi}\gamma)\lambda + ia_2(\bar{\Psi}\gamma)\chi + a_3\bar{\lambda}\chi + a_4\bar{\chi}\chi - \\ & -\frac{1}{2}\bar{\Omega}_\mu\sigma^{\mu\nu}\Omega_\nu + ib_1(\bar{\Omega}\gamma)\rho + ib_2(\bar{\Omega}\gamma)\psi + b_3\bar{\rho}\psi + b_4\bar{\psi}\psi \end{aligned} \tag{8.4}$$

Indeed, the invariance of the total Lagrangian under the (corrected) supertransformations could be achieved provided:

$$a_1 = b_1 = \frac{1}{\sqrt{2}}, \qquad a_2 = b_2 = 1, \qquad a_3 = b_3 = -\sqrt{2}, \qquad a_4 = b_4 = -\frac{1}{2}$$

As in all previous cases, we have local gauge symmetries corresponding to two spin 3/2 particles:

$$\begin{aligned} \delta\Psi_\mu &= \partial_\mu \xi_1 + \frac{im}{2}\gamma_\mu \xi_1 \qquad \delta\lambda = \frac{m}{\sqrt{2}}\xi_1 \qquad \delta\chi = m\xi_1 \\ \delta\Omega_\mu &= \partial_\mu \xi_2 + \frac{im}{2}\gamma_\mu \xi_2 \qquad \delta\rho = \frac{m}{\sqrt{2}}\xi_2 \qquad \delta\psi = m\xi_2 \end{aligned} \tag{8.5}$$

In this, with the help of these transformations, introducing gauge invariant derivative $D_\mu z_i = \partial_\mu z_i - mC_{\mu i}$ we obtain final form of fermionic supertransformations:

$$\begin{aligned} \delta\Psi_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta i}\varepsilon^{ij}\eta_j + D_\mu z_i \varepsilon^{ij}\eta_j \\ \delta\Omega_\mu &= -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu \bar{C}_{\alpha\beta}{}^i\eta_i + D_\mu \bar{z}^i \eta_i \\ \delta\lambda &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}C_{\alpha\beta i}\varepsilon^{ij}\eta_j \qquad \delta\chi = -i\hat{D}z_i\varepsilon^{ij}\eta_j \\ \delta\rho &= -\frac{1}{2\sqrt{2}}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta}{}^i\eta_i \qquad \delta\psi = -i\hat{D}\bar{z}^i\eta_i \end{aligned} \tag{8.6}$$

Such supermultiplet could appear when $N = 4$ supergravity is broken up to $N = 2$ and two massive gravitini have equal masses [40, 42, 43].

9. $N = 4$ supermultiplet with central charge

Our last example — massive $N = 4$ supermultiplet with central charge. Such multiplets contains twice as many fields as massive $N = 2$ supermultiplet without central charge and in the massless limit it gives just two massless spin 3/2 supermultiplets:

$$2 \otimes \begin{pmatrix} \frac{3}{2} \\ 4 \otimes 1 \\ 6 \otimes \frac{1}{2} \\ 4 \otimes 0 \end{pmatrix} \quad \Rightarrow \quad 2 \otimes \begin{pmatrix} \frac{3}{2} \\ 4 \otimes 1 \\ 7 \otimes \frac{1}{2} \\ 8 \otimes 0 \end{pmatrix}$$

This time even using the usual trick with vector fields it is impossible to obtain complete SU(4) symmetry. Indeed [14] maximum symmetry that we can get here is the $USp(4)$ one. Thus we introduce $USp(4)$ invariant antisummetric tensor $\omega_{[ij]}$ such that $\omega^{ij}\omega_{jk} = -\delta^i{}_k$ and use it to construct $USp(4)$ invariant form of supertransformations for massless supermultiplets:

$$\delta\Psi_\mu = -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta i}\omega^{ij}\eta_j \qquad \delta\Omega_\mu = -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu \bar{C}_{\alpha\beta}{}^i\eta_i$$
$$\delta\bar{C}_\mu{}^i = 2(\bar{\Psi}_\mu\omega^{ij}\eta_j) + 2i(\bar{\rho}^{ij}\gamma_\mu\eta_j) \qquad \delta C_\mu{}^i = 2(\bar{\Omega}_\mu\eta_i) + 2i(\bar{\lambda}_{ij}\gamma_\mu\omega^{jk}\eta_k)$$
$$\delta\lambda^{ij} = -\frac{1}{2}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta}{}^{[i}\omega^{j]k}\eta_k - i\sqrt{2}\hat{\partial}z_{[i}\eta_{j]} + \frac{i}{\sqrt{2}}\omega^{ij}\hat{\partial}z_k\omega^{kl}\eta_l \qquad \delta\chi = -i\hat{\partial}\bar{z}^i\eta_i \tag{9.1}$$
$$\delta\rho_{ij} = -\frac{1}{2}\sigma^{\alpha\beta}C_{\alpha\beta[i}\eta_{j]} - i\sqrt{2}\hat{\partial}\bar{\Phi}^{[i}\omega^{j]k}\eta_k - \frac{i}{\sqrt{2}}\omega_{ij}\hat{\partial}\bar{\Phi}^k\eta_k \qquad \delta\psi = -i\hat{\partial}\Phi_i\omega^{ij}\eta_j$$
$$\delta\bar{z}^i = 2\sqrt{2}(\bar{\lambda}^{ij}\eta_j) - \sqrt{2}(\bar{\lambda}^{kl}\omega_{kl}\omega^{ij}\eta_j) \qquad \delta z_i = 2(\bar{\chi}\eta_i)$$
$$\delta\Phi_i = 2\sqrt{2}(\bar{\rho}_{ij}\omega^{jk}\eta_k) + \sqrt{2}(\bar{\rho}_{kl}\omega^{kl}\eta_i) \qquad \delta\bar{\Phi}^i = 2(\bar{\psi}\omega^{ij}\eta_j)$$

Then subsequent calculations lead us to the following mass terms for bosons:

$$\mathcal{L}_b = \frac{m}{2\sqrt{2}}C_{\mu i}\omega^{ij}\partial_\mu(z_j - \Phi_j) + h.c. + \frac{m^2}{2}\bar{C}_\mu{}^i C_{\mu i} - \frac{m^2}{4}(\bar{z}^i + \bar{\Phi}^i)(z_i + \Phi_i) \tag{9.2}$$

from which we see that combination $z_i - \Phi_i$ plays the role of Goldtone fields while $z_i + \Phi_i$ remains as physical massive scalars. As for the fermionic fields, their mass terms turn out to be:

$$\begin{aligned}\frac{1}{m}\mathcal{L} = & -\frac{1}{2}\bar{\Psi}_\mu\sigma^{\mu\nu}\Psi_\nu + \frac{i}{2}(\bar{\Psi}\gamma)\rho - \frac{i}{\sqrt{2}}(\bar{\Psi}\gamma)\chi + \frac{1}{2}\omega^{ik}\omega^{jl}\bar{\rho}_{ij}\rho_{kl} - \frac{1}{4}\bar{\rho}\rho + \frac{1}{\sqrt{2}}\bar{\rho}\chi - \\ & -\frac{1}{2}\bar{\Omega}_\mu\sigma^{\mu\nu}\Omega_\nu - \frac{i}{2}(\bar{\Omega}\gamma)\lambda - \frac{i}{\sqrt{2}}(\bar{\Omega}\gamma)\psi + \frac{1}{2}\omega_{ik}\omega_{jl}\bar{\lambda}^{ij}\lambda^{kl} - \frac{1}{4}\bar{\lambda}\lambda - \frac{1}{\sqrt{2}}\bar{\lambda}\psi\end{aligned} \tag{9.3}$$

which corresponds to invariance under the following two local gauge transformations:

$$\delta\Psi_\mu = \partial_\mu\xi_1 + \frac{im}{2}\gamma_\mu\xi_1, \qquad \delta\rho_{ij} = \frac{m}{2}\omega_{ij}\xi_1, \qquad \delta\chi = -\frac{m}{\sqrt{2}}\xi_1$$
$$\delta\Omega_\mu = \partial_\mu\xi_2 + \frac{im}{2}\gamma_\mu\xi_2, \qquad \delta\lambda^{ij} = -\frac{m}{2}\omega^{ij}\xi_2, \qquad \delta\psi = -\frac{m}{\sqrt{2}}\xi_2$$

With the help of these transformations and introducing gauge invariant objects:

$$D_\mu z_i = \partial_\mu z_i - \frac{m}{\sqrt{2}}\omega_{ij}\bar{C}_\mu{}^j, \qquad D_\mu\Phi_i = \partial_\mu\Phi_i + \frac{m}{\sqrt{2}}\omega_{ij}\bar{C}_\mu{}^j$$

we obtain final form of fermionic supertransformations:

$$\delta\Psi_\mu = -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu C_{\alpha\beta i}\omega^{ij}\eta_j - \frac{1}{\sqrt{2}}D_\mu(\bar{z}^i - \bar{\Phi}^i)\eta_i$$
$$\delta\Omega_\mu = -\frac{i}{4}\sigma^{\alpha\beta}\gamma_\mu\bar{C}_{\alpha\beta}{}^i\eta_i + \frac{1}{\sqrt{2}}D_\mu(z_i - \Phi_i)\omega^{ij}\eta_j$$
$$\begin{aligned}\delta\lambda^{ij} = & -\frac{1}{2}\sigma^{\alpha\beta}\bar{C}_{\alpha\beta}{}^{[i}\omega^{j]k}\eta_k - i\sqrt{2}\hat{D}z_{[i}\eta_{j]} + \frac{i}{\sqrt{2}}\omega^{ij}\hat{D}z_k\omega^{kl}\eta_l - \\ & -\frac{m}{\sqrt{2}}[(z_k + \Phi_k)\omega^{k[i}\omega^{j]l}\eta_l + \frac{1}{2}\omega^{ij}(z_k + \Phi_k)\omega^{kl}\eta_l]\end{aligned}$$
$$\delta\rho_{ij} = -\frac{1}{2}\sigma^{\alpha\beta}C_{\alpha\beta[i}\eta_{j]} - i\sqrt{2}\hat{D}\bar{\Phi}^{[i}\omega^{j]k}\eta_k - \frac{i}{\sqrt{2}}\omega_{ij}\hat{D}\bar{\Phi}^k\eta_k + \frac{m}{\sqrt{2}}[(\bar{z}^k + \bar{\Phi}^k)\omega_{k[i}\eta_{j]} + \omega_{ij}(\bar{z}^k + \bar{\Phi}^k)\eta_k]$$
$$\delta\chi = -i\hat{D}\bar{z}^i\eta_i + \frac{m}{2}(\bar{z}^i + \bar{\Phi}^i)\eta_i$$
$$\delta\psi = -i\hat{D}\Phi_i\omega^{ij}\eta_j + \frac{m}{2}(z_i + \Phi_i)\omega^{ij}\eta_j$$

10. Conclusion

Thus we give explicit construction of massive spin 3/2 supermultiplets out of the massless ones and this gives us important and model independent information om the structure of supergravity models where such supermultiplets could arise as a result of spontaneous supersymmetry breaking. Also we hope that experience gained will be helpful in investigation of massive supermultiplets with arbitrary superspins.

References

[1] Y.M. Zinoviev *Gauge invariant description of massive high spin particles*, Preprint 83-91, IHEP, Protvino (1983).

[2] S.M. Klishevich and Y.M. Zinovev, *Electromagnetic interaction of a massive spin-2 particle*, *Phys. Atom. Nucl.* **61** (1998) 1527 [hep-th/9708150].

[3] Y.M. Zinoviev, *On massive high spin particles in (A)dS*, hep-th/0108192.

[4] N. Arkani-Hamed, H. Georgi and M.D. Schwartz, *Effective field theory for massive gravitons and gravity in theory space*, *Ann. Phys. (NY)* **305** (2003) 96 [hep-th/0210184].

[5] S. Hamamoto, *Possible nonlinear completion of massive gravity*, *Prog. Theor. Phys.* **114** (2006) 1261 [hep-th/0505194].

[6] M. Bianchi, P.J. Heslop and F. Riccioni, *More on la grande bouffe*, *JHEP* **08** (2005) 088 [hep-th/0504156].

[7] K. Hallowell and A. Waldron, *Constant curvature algebras and higher spin action generating functions*, *Nucl. Phys.* **B 724** (2005) 453 [hep-th/0505255].

[8] I.L. Buchbinder and V.A. Krykhtin, *Gauge invariant lagrangian construction for massive bosonic higher spin fields in D dimensions*, *Nucl. Phys.* **B 727** (2005) 537 [hep-th/0505092].

[9] I.L. Buchbinder, V.A. Krykhtin and P.M. Lavrov, *Gauge invariant lagrangian formulation of higher spin massive bosonic field theory in AdS space*, *Nucl. Phys.* **B 762** (2007) 344 [hep-th/0608005].

[10] R.R. Metsaev, *Gauge invariant formulation of massive totally symmetric fermionic fields in (A)dS space*, *Phys. Lett.* **B 643** (2006) 205 [hep-th/0609029].

[11] Y.M. Zinoviev, *On massive spin 2 interactions*, *Nucl. Phys.* **B 770** (2007) 83 [hep-th/0609170].

[12] R.R. Metsaev, *Gravitational and higher-derivative interactions of massive spin 5/2 field in (A)dS space*, hep-th/0612279.

[13] I.L. Buchbinder, V.A. Krykhtin and A.A. Reshetnyak, *Brst approach to lagrangian construction for fermionic higher spin fields in AdS space*, hep-th/0703049.

[14] L. Andrianopoli, R. D'Auria, S. Ferrara and M.A. Lledo, *Super Higgs effect in extended supergravity*, *Nucl. Phys.* **B 640** (2002) 46 [hep-th/0202116].

[15] S. Ferrara and P. van Nieuwenhuizen, *Noether coupling of massive gravitinos to $N = 1$ supergravity*, *Phys. Lett.* **B 127** (1983) 70.

[16] Y.M. Zinoviev, *Spontaneous symmetry breaking in $N = 2$ supergravity*, *Yad. Fiz.* **46** (1987) 943.

[17] I.L. Buchbinder, S.J. Gates Jr., I. Linch, W. Divine and J. Phillips, *Dynamical superfield theory of free massive superspin-1 multiplet*, *Phys. Lett.* **B 549** (2002) 229 [hep-th/0207243].

[18] I.L. Buchbinder, J. James Gates, S., S.M. Kuzenko and J. Phillips, *Massive 4D, $N = 1$ superspin 1 and $3/2$ multiplets and dualities*, *JHEP* **02** (2005) 056 [hep-th/0501199].

[19] S. Deser and A. Waldron, *Gauge invariances and phases of massive higher spins in (A)dS*, *Phys. Rev. Lett.* **87** (2001) 031601 [hep-th/0102166].

[20] S. Deser and A. Waldron, *Partial masslessness of higher spins in (A)dS*, *Nucl. Phys.* **B 607** (2001) 577 [hep-th/0103198].

[21] S. Deser and A. Waldron, *Null propagation of partially massless higher spins in (A)dS and cosmological constant speculations*, *Phys. Lett.* **B 513** (2001) 137 [hep-th/0105181].

[22] R. Altendorfer and J. Bagger, *Dual anti-de Sitter superalgebras from partial supersymmetry breaking*, *Phys. Rev.* **D 61** (2000) 104004 [hep-th/9908084].

[23] Y.M. Zinoviev, *Massive spin-2 supermultiplets*, hep-th/0206209.

[24] I.L. Buchbinder, S.J. Gates Jr., I. Linch, William Divine and J. Phillips, *New 4D, $N = 1$ superfield theory: model of free massive superspin-$3/2$ multiplet*, *Phys. Lett.* **B 535** (2002) 280 [hep-th/0201096].

[25] S.J. Gates Jr., S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *New massive supergravity multiplets*, *JHEP* **02** (2007) 052 [hep-th/0610333].

[26] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, *The $m \to 0$ limit for massive graviton in dS_4 and AdS_4: how to circumvent the van Dam-Veltman-Zakharov discontinuity*, *Phys. Lett.* **B 503** (2001) 173 [hep-th/0011138].

[27] M. Porrati, *No Van Dam-Veltman-Zakharov discontinuity in AdS space*, *Phys. Lett.* **B 498** (2001) 92 [hep-th/0011152].

[28] C. Deffayet, G.R. Dvali, G. Gabadadze and A.I. Vainshtein, *Nonperturbative continuity in graviton mass versus perturbative discontinuity*, *Phys. Rev.* **D 65** (2002) 044026 [hep-th/0106001].

[29] P.A. Grassi and P. van Nieuwenhuizen, *No van Dam-Veltman-Zakharov discontinuity for supergravity in AdS space*, *Phys. Lett.* **B 499** (2001) 174 [hep-th/0011278].

[30] S. Deser and A. Waldron, *(Dis)continuities of massless limits in spin $3/2$-mediated interactions and cosmological supergravity*, *Phys. Lett.* **B 501** (2001) 134 [hep-th/0012014].

[31] M.J. Duff, J.T. Liu and H. Sati, *Quantum discontinuity for massive spin $3/2$ with a lambda term*, *Nucl. Phys.* **B 680** (2004) 117 [hep-th/0211183].

[32] S. Cecotti, L. Girardello and M. Porrati, *An exceptional $N = 2$ supergravity with flat potential and partial superhiggs*, *Phys. Lett.* **B 168** (1986) 83.

[33] Y.M. Zinoviev, *Spontaneous symmetry breaking in $N = 2$ supergravity with matter*, *Int. J. Mod. Phys.* **A 7** (1992) 7515.

[34] Y.M. Zinovev, *Dual versions of $N = 2$ supergravity and spontaneous supersymmetry breaking*, *Class. and Quant. Grav.* **12** (1995) 1355 [hep-th/9409199].

[35] L. Andrianopoli, R. D'Auria, S. Ferrara and M.A. Lledo, *$N = 2$ super-higgs, $N = 1$ Poincaré vacua and quaternionic geometry*, *JHEP* **01** (2003) 045 [hep-th/0212236].

[36] T. Garidi, *What is mass in de Sitterian physics?*, hep-th/0309104.

[37] E.A. Ivanov, A.S. Sorin, *Superfield formulation of OSp(1,4) supersymmetry*, *J. Phys.* **A 13** (1980) 1159.

[38] T. Gregoire, M.D. Schwartz and Y. Shadmi, *Massive supergravity and deconstruction*, *JHEP* **07** (2004) 029 [hep-th/0403224].

[39] Y.M. Zinoviev, *Spontaneous symmetry breaking in $N = 3$ supergravity*, Preprint 86-171, IHEP, Serpukhov (1986).

[40] Y.M. Zinoviev, *Spontaneous symmetry breaking in $N = 4$ supergravity*, Preprint 87-101, IHEP, Serpukhov (1987).

[41] V.A. Tsokur and Y.M. Zinoviev, *Spontaneous supersymmetry breaking in $N = 3$ supergravity with matter*, *Yad. Fiz.* **59** (1996) 1169.

[42] V.A. Tsokur and Y.M. Zinoviev, *Spontaneous supersymmetry breaking in $N = 4$ supergravity with matter*, *Yad. Fiz.* **59** (1996) 2277.

[43] R. D'Auria, S. Ferrara and S. Vaula, *N = 4 gauged supergravity and a IIB orientifold with fluxes*, *New J. Phys.* **4** (2002) 71 [hep-th/0206241].

[44] D. Fabbri and P. Fre, *Shadow multiplets and superhiggs mechanism*, *Fortschr. Phys.* **49** (2001) 475 [hep-th/0101050].